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# Time Evolution of Husimi Function for Photon-Added Squeezed Vacuum State in Dissipative Channel

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**Abstract** In this paper, we determine the *normalization factor of photo-added squeezed vacuum state (PASVS)* as a Legendre polynomial of the squeezing parameter by virtue of principle of mathematical induction, and derive the normally ordered density operator of PASVS in dissipative channel. In addition, we also give the explicit analytical expression of Husimi function in dissipative channel and study its behavior with evolution time graphically.

**Keywords** Photon-added squeezed vacuum state · Principle of mathematical induction · Husimi function · Dissipative channel

## 1 Introduction

Nonclassical optical fields plays a crucial role in understanding fundaments of quantum physics and have many applications in quantum information processing [1]. Usually, the nonclassicality manifests itself in specific properties of quantum statistics, such as the sub-Poissonian photon statistics [2], squeezing in one of the quadratures of the field [3], anti-bunching [4], and negativity of Wigner function (WF) [5], etc. Especially, the non-positive WF is regarded as a clear signature of the highly nonclassical character of the optical fields [6] and is often used to investigate the decoherence of quantum states [7, 8]. However, the WF itself is not a probability distribution due to being not always positive. To overcome this shortcoming, the so-called Husimi function (HF) is introduced [9–14], which is defined in a manner that guarantees it to be non-negative and gives it a probability interpretation. The Husimi distribution is a mathematical tool used in physics, and was introduced in 1940 [15]. The Husimi representation is a quasi-probability distribution commonly used in quantum mechanics and also to represent the quantum state of light [16]. It is used in the field of quantum optics and particularly for tomographic purposes. It is also applied in the study of

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quantum effects in superconductors [17]. The Husimi distribution is the simplest distribution of quasi-probability in phase space [18]. In addition, the Husimi function can present its more useful value for dealing with the nonlinear problem of quantum dynamics. The theoretical calculation of HF can help experimentalists to judge the quality of the experiment.

Recently, it has been realized that adding photons to a state is an important approach to obtaining new nonclassical states. For example, photon-added coherent state (PACS), first introduced by Agarwal and Tara [19], is an intermediate state between the Fock state and the coherent state. PACS exhibits both higher-order squeezing and higher-order sub-Poissonian character [20]. The single and two PACSs were prepared experimentally by Zavatta's group [21] and Kalamidas' group [22], respectively. As another example, photo-added squeezed vacuum state (PASVS), called excited squeezed vacuum state as well, is of great interest because it displays strong nonclassical properties [23]. Its WF and tomogram is derived by using the coherent state representation of Wigner operator and the intermediate coordinate-momentum representation [24]. As is well known, dissipative quantum channels tend to deteriorate the degree of nonclassicality. Thus, it is necessary to investigate the dynamical behaviors of HF in dissipative channels.

In the present paper, we focus our study on the decoherence of PASVS in dissipative channel by describing the time evolution of its HF. Our work lies on as follows: (i) Through using the principle of mathematical induction, PASVS is normalized in the form of Legendre polynomial of the squeezing parameter. (ii) We derive the normal ordered form of PASVS's density operator in dissipative channel, which is very convenient to calculate the HF. (iii) The explicit analytical expression of HF is deduced in dissipative channel. We find that the HF losses its non-Gaussian nature and become Gaussian at long times by virtue of studying the behavior of HF evolution graphically.

## 2 Normalization of PASVS

In Refs. [23, 24], the mathematical and physical properties of PASVS have been discussed in detail. Here, we derive the normalization factor of PASVS by using the principle of mathematical induction. Theoretically, PASVS can be obtained by repeatedly operating the photon creation operator $a^\dagger$ on a squeezed vacuum state $S(r)|0\rangle$, i.e.,

$$|r,m\rangle = N_{r,m} a^{\dagger m} S(r)|0\rangle, \tag{1}$$

where $|0\rangle$ is single mode vacuum, $S(r)$ is the single-mode squeezing operator $S(r) = \exp[r(a^{\dagger 2} - a^2)/2]$ with $r$ being the squeezing parameter, $[a, a^\dagger] = 1$, and $N_{r,m}$ is the normalization constant to be determined.

In order to obtain the normalization factor, we let

$$|r\rangle_m \equiv a^{\dagger m} S(r)|0\rangle, \tag{2}$$

where $|r\rangle_0 \equiv |r\rangle = S(r)|0\rangle = \sqrt{\operatorname{sech} r}\exp(\frac{a^{\dagger 2}}{2}\tanh r)|0\rangle$. According to principle of mathematical induction and $a|r\rangle = a^\dagger \tanh r|r\rangle$, we have

$$\begin{aligned} {}_1\langle r|r\rangle_1 &= \tanh^2 r \operatorname{sech} r \langle 0| \exp\left(\frac{a^2}{2}\tanh r\right) a a^\dagger \exp\left(\frac{a^{\dagger 2}}{2}\tanh r\right)|0\rangle + 1 \\ &= \cosh r\, P_1(\cosh r), \end{aligned} \tag{3}$$

where $P_1(x) = x$ is the first-order Legendre polynomial, and

$$\begin{aligned} {}_2\langle r|r\rangle_2 &= \langle r|(a^\dagger a+1)(a^\dagger a+1)|r\rangle +_1 \langle r|r\rangle_1 \\ &= 2\cosh^2 r\, P_2(\cosh r), \end{aligned}$$

where $P_2(x) = \frac{1}{2}(3x^2 - 1)$ is the second-order Legendre polynomial. Similarly, for cases $n \le m$, it is easily obtained the following expression

$$ {}_{n-1}\langle r|r\rangle_{n-1} = (\cosh r)^{n-1}(n-1)!P_{n-1}(\cosh r), \tag{4}$$

where $P_m(x)$ is $m$-order Legendre polynomial whose definition is

$$P_m(x) = \sum_{l=0}^{[m/2]} \frac{(2m-2l)!(-1)^l x^{m-2l}}{2^m l!(m-l)!(m-2l)!}. \tag{5}$$

Therefore, we can also prove that

$$\begin{aligned} {}_m\langle r|r\rangle_m &= \langle r|a^{m-1}(a^\dagger a+1)a^{\dagger m-1}|r\rangle \\ &= \tanh^2 r\, {}_m\langle r|r\rangle_m + (2m-1)_{m-1}\langle r|r\rangle_{m-1} - (m-1)^2_{m-2}\langle r|r\rangle_{m-2}, \end{aligned} \tag{6}$$

i.e.,

$$ {}_m\langle r|r\rangle_m = \cosh^2 r[(2m-1)_{m-1}\langle r|r\rangle_{m-1} - (m-1)^2_{m-2}\langle r|r\rangle_{m-2}]. \tag{7}$$

Comparing (7) with the recurrence relation of Legendre polynomial

$$(m+1)P_{m+1}(x) - (2m+1)xP_m(x) + mP_{m-1}(x) = 0, \tag{8}$$

we have

$$ {}_m\langle r|r\rangle_m = m!\cosh^m r\, P_m(\cosh r), \tag{9}$$

so the compact expression for the normalization factor of PASVS is

$$N_{r,m}^{-2} = m!\cosh^m r\, P_m(\cosh r), \tag{10}$$

which is very useful for the following calculation. From (10) it is clearly seen that PASVS's normalization factor is related to an $m$-order Legendre polynomial of the squeezing parameter $r$, where $m$ is just the number of added photons.

## 3 Normal Ordered Form of PASVS's Density Operator in Dissipative Channel

When the PASVS evolves in the dissipative channel (or cavity at zero temperature), the evolution of the density matrix can be described by [25]

$$\frac{d\rho(t)}{dt} = \kappa[2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a], \tag{11}$$

where $[a, a^\dagger] = 1$ and $\kappa$ is the dissipative coefficient. Fortunately, by virtue of the entangled state representation, the density operator $\rho(t)$ for the dissipative channel satisfies the following relation

$$\rho(t) = \sum_{n=0}^{\infty} \frac{T^n}{n!} e^{-\kappa t a^\dagger a} a^n \rho_0 a^{\dagger n} e^{-\kappa t a^\dagger a}, \tag{12}$$

where $T = 1 - e^{-2\kappa t}$, which is just the Kraus operator sum representation. For more discussion, the readers may see Ref. [26].

As the initial density matrix $\rho_0$ is the PASVS in (12), namely,

$$\rho_0 = |r, m\rangle\langle r, m| \equiv N_{r,m}^2 a^{\dagger m} S(r)|0\rangle\langle 0|S^\dagger(r)a^m \tag{13}$$

substituting it into (12) yields

$$\rho_{r,m}(t) = \sum_{n=0}^{\infty} \frac{T^n}{n!} G_{r,m,n}(a, a^\dagger; t), \tag{14}$$

where

$$G_{r,m,n}(a, a^\dagger; t) = N_{r,m}^2 e^{-\kappa t a^\dagger a} a^n a^{\dagger m} S(r)|0\rangle\langle 0|S^\dagger(r)a^m a^{\dagger n} e^{-\kappa t a^\dagger a}. \tag{15}$$

By noticing

$$S(r)|0\rangle = \operatorname{sech}^{1/2} r \exp\left(\frac{\tanh r}{2} a^{\dagger 2}\right)|0\rangle \tag{16}$$

and using

$$e^{-\kappa t a^\dagger a} a e^{\kappa t a^\dagger a} = a e^{\kappa t}, \qquad e^{-\kappa t a^\dagger a} a^\dagger e^{\kappa t a^\dagger a} = a^\dagger e^{-\kappa t}, \tag{17}$$

(15) can re-expressed as

$$G_{r,m,n}(a, a^\dagger; t) = N_{r,m}^2 \operatorname{sech} r\, e^{2(n-m)\kappa t} a^n a^{\dagger m} \exp(\mu^2 a^{\dagger 2})|0\rangle\langle 0| \exp(\mu^2 a^2) a^m a^{\dagger n}, \tag{18}$$

where we have set $\mu = \sqrt{\frac{\tanh r}{2}} e^{-\kappa t}$.

On the other hand, using the completeness relation of coherent state and the integral formula

$$H_{m,n}(\xi, \zeta) = (-1)^n e^{\xi\zeta} \int \frac{d^2 z}{\pi} z^n z^{*m} \exp\left(-|z|^2 + \xi z - \zeta z^*\right), \tag{19}$$

where $H_{m,n}(\xi, \zeta)$ is two-variable Hermite polynomial

$$H_{m,n}(\xi, \zeta) = \sum_{l=0}^{\min(m,n)} \frac{m!n!(-1)^l}{(n-l)!(m-l)!} \frac{\xi^{m-l}\zeta^{n-l}}{l!}, \tag{20}$$

we obtain the normal ordering form of Bose operator $a^n a^{\dagger m}$ as follows ($:\,:$ denotes the normal ordering)

$$a^n a^{\dagger m} = \int \frac{d^2 z}{\pi} a^n |z\rangle\langle z| a^{\dagger m} = (-i)^{n+m} : H_{m,n}(ia^\dagger, ia):. \tag{21}$$

It follows that

$$a^n a^{\dagger m} \exp(\mu^2 a^{\dagger 2})|0\rangle = \sum_{l=0}^{\infty} \frac{\mu^{2l}}{l!} (-i)^{2l+m+n} H_{2l+m,n}(ia^\dagger, 0)|0\rangle, \tag{22}$$

and

$$\langle 0|\exp(\mu^2 a^2)a^m a^{\dagger n} = \sum_{k=0}^{\infty}\frac{\mu^{2k}}{k!}(-i)^{2k+m+n}\langle 0|H_{n,2k+m}(0,ia). \tag{23}$$

Putting (22) and (23) into (18) and considering the normal product form of vacuum projector $|0\rangle\langle 0| =: \exp(-a^\dagger a):$, (18) is simplified as

$$G_{r,m,n}(a,a^\dagger;t) = N_{r,m}^2 e^{2(n-m)\kappa t}\operatorname{sech} r\sum_{k,l=0}^{\infty}\frac{\mu^{2k+2l}(2l+m)!(2k+m)!}{k!l!(2l+m-n)!(2k+m-n)!}$$
$$\times :a^{\dagger 2l+m-n}e^{-a^\dagger a}a^{2k+m-n}:. \tag{24}$$

Here we have used

$$H_{m,n}(\xi,0) = \frac{m!(-1)^n}{(m-n)!}\xi^{m-n}, \qquad H_{m,n}(0,\zeta) = \frac{n!(-1)^m}{(n-m)!}\zeta^{n-m} \tag{25}$$

which is easily obtained from the definition of (20). Note that the condition $n \leqslant m$ must satisfy for (24). Then

$$\rho_{r,m}(t) = \sum_{n=0}^{m}\frac{T^n}{n!}G_{r,m,n}(a,a^\dagger;t). \tag{26}$$

Equation (26) is just the normal ordered form of PASVS's density operator in dissipative channel, which is very convenient to calculate the following different phase space distributions.

Thus, as the result of (24) and (26), when $\kappa t \to \infty$, $T = 1$, we find $\rho_{r,m}(\infty) \to |0\rangle\langle 0|$, which implies that the system state reduces to a Gaussian state after a long time interaction in the channel.

## 4 Husimi Function of $\rho_{r,m}(t)$

In this section, we discuss the HF of PASVS and its effect with time evolution in dissipative channel. To begin with, we derive the expression of HF evolution of $\rho_{r,m}(t)$ for PASVS in dissipative channel. Recall that Husimi operator $\Delta_h(q,p;g)$ by smoothing out the Wigner operator $\Delta_w(q',p')$ is introduced via averaging over a "coarse graining" function [27]

$$\Delta_h(q,p;g) = 2\int \mathrm{d}q'\,\mathrm{d}p'\,\Delta_w(q',p')e^{-g(q'-q)^2-\frac{1}{g}(p'-p)^2}, \tag{27}$$

where positive $g$ is the Gaussian spatial width parameter, and may express a pure state density operator, i.e.,

$$\Delta_h(q,p;g) = |q,p\rangle_{g,g}\langle q,p|, \tag{28}$$

where $|q,p\rangle_g$ is expressed as

$$|q,p\rangle_g = \sqrt{C}\exp\left(\frac{D}{2} + A^* a^\dagger + B a^{\dagger 2}\right)|0\rangle \tag{29}$$

with $A = \frac{\sqrt{2}(gq-ip)}{1+g}$, $B = \frac{1-g}{2(1+g)}$, $C = \frac{2\sqrt{g}}{1+g}$, $D = -\frac{gq^2+p^2}{1+g}$. Thus, the HF of various density matrix is given by

$$\mathcal{H}(p,q) \equiv {}_g\langle q,p|\rho|q,p\rangle_g, \tag{30}$$

which is a pure state expectation value. For simplicity, here we assume $0 < g < 1$. Equation (30) provides us a good representative space for studying various properties of HF.

Substituting (24) and (26) into (30), the HF of $\rho_{r,m}(t)$ for PASVS is described as

$$\begin{aligned}\mathcal{H}(p,q;t) = N_{r,m}^2 \operatorname{sech} r \sum_{k,l=0}^{\infty}\sum_{n=0}^{m} \frac{\mu^{2k+2l}(2l+m)!(2k+m)!T^n e^{2(n-m)\kappa t}}{n!k!l!(2l+m-n)!(2k+m-n)!} \\ \times {}_g\langle q,p|a^{\dagger 2l+m-n}|0\rangle\langle 0|a^{2k+m-n}|q,p\rangle_g.\end{aligned} \tag{31}$$

Inserting the completeness relation of coherent state, we have

$$\begin{aligned}&{}_g\langle q,p|a^{\dagger 2l+m-n}|0\rangle\langle 0|a^{2k+m-n}|q,p\rangle_g \\ &= {}_g\langle q,p|\int\frac{d^2z_1}{\pi}|z_1\rangle\langle z_1|a^{\dagger 2l+m-n}|0\rangle\langle 0|a^{2k+m-n}\int\frac{d^2z_2}{\pi}|z_2\rangle\langle z_2||q,p\rangle_g \\ &= Ce^D\int\frac{d^2z_1}{\pi}z_1^{*2l+m-n}e^{-|z_1|^2+Az_1+Bz_1^2}\int\frac{d^2z_2}{\pi}z_2^{2k+m-n}e^{-|z_2|^2+A^*z_2^*+Bz_2^{*2}} \\ &= Ce^D\frac{\partial^{2l+m-n}}{\partial\varepsilon^{2l+m-n}}\int\frac{d^2z_1}{\pi}e^{-|z_1|^2+Az_1+\varepsilon z_1^*+Bz_1^2}|_{\varepsilon=0} \\ &\quad\times\frac{\partial^{2k+m-n}}{\partial\nu^{2k+m-n}}\int\frac{d^2z_2}{\pi}e^{-|z_2|^2+\nu z_2+A^*z_2^*+Bz_2^{*2}}|_{\nu=0}.\end{aligned} \tag{32}$$

Using the following integral formula [28]

$$\int\frac{d^2z}{\pi}e^{\zeta|z|^2+\xi z+\eta z^*+fz^2+gz^{*2}} = \frac{1}{\sqrt{\zeta^2-4fg}}e^{\frac{-\zeta\xi\eta+\xi^2g+\eta^2f}{\zeta^2-4fg}}, \tag{33}$$

whose convergent condition is $\mathtt{Re}(\zeta\pm f\pm g) < 0$ and $\mathtt{Re}(\frac{\zeta^2-4fg}{\zeta\pm f\pm g}) < 0$, (32) is calculated as

$$\begin{aligned}&{}_g\langle q,p|a^{\dagger 2l+m-n}|0\rangle\langle 0|a^{2k+m-n}|q,p\rangle_g \\ &= Ce^D\left(\frac{\partial^{2l+m-n}}{\partial\varepsilon^{2l+m-n}}e^{A\varepsilon+\varepsilon^2B}\right)\bigg|_{\varepsilon=0}\left(\frac{\partial^{2k+m-n}}{\partial\nu^{2k+m-n}}e^{\nu A^*+\nu^2B}\right)\bigg|_{\nu=0} \\ &= Ce^D(-1)^{l+k}B^{l+m-n+k}H_{2l+m-n}\left(\frac{A}{2i\sqrt{B}}\right)H_{2k+m-n}\left(-\frac{A^*}{2i\sqrt{B}}\right).\end{aligned} \tag{34}$$

Note that we have considered the generating function of Hermite polynomial $H_m(x)$,

$$H_s(x) = \frac{\partial^s}{\partial t^s}e^{2tx-t^2}|_{t=0}, \qquad H_s(x) = \sum_{h=0}^{[s/2]}\frac{(-1)^h s!}{h!(s-2h)!}(2x)^{s-2h}. \tag{35}$$

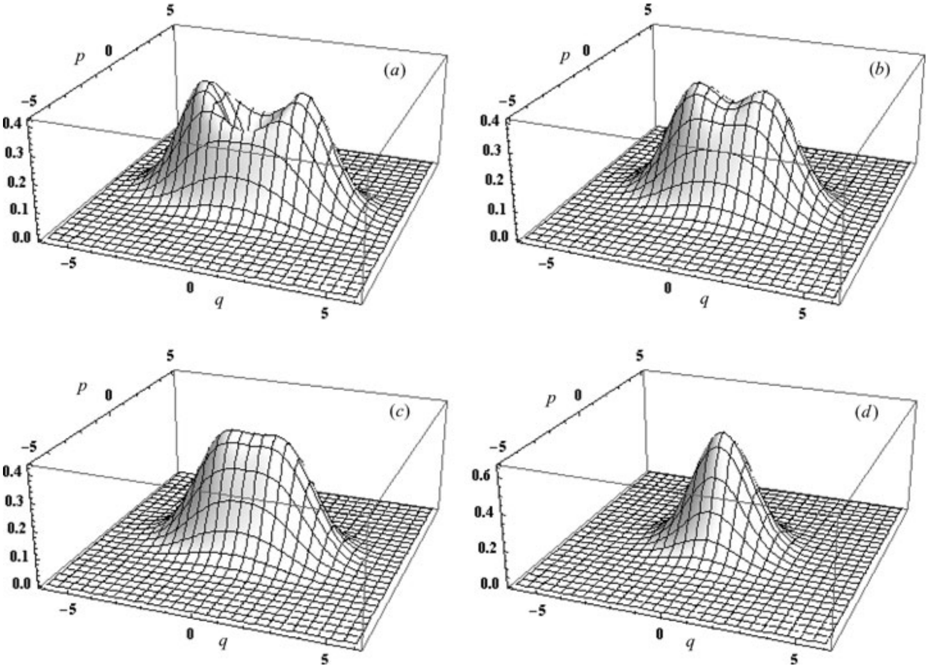


**Fig. 1** HF of PASVS with $m=1$, $r=0.5$, and $g=0.5$ for the different evolution time: **(a)** $\kappa t=0.05$, **(b)** $\kappa t=0.25$, **(c)** $\kappa t=0.35$, and **(d)** $\kappa t=5$, respectively

Substituting (34) into (31), then the final form of HF evolution for PASVS in dissipative channel is

$$\mathcal{H}(p,q;t)=N_{r,m}^{2}Ce^{D}\operatorname{sech}r\sum_{n=0}^{m}\frac{T^{n}e^{2(n-m)\kappa t}B^{m-n}}{n!}\times\left|\sum_{l=0}^{\infty}\frac{\mu^{2l}(2l+m)!(-B)^{l}}{l!(2l+m-n)!}H_{2l+m-n}\left(\frac{A}{2i\sqrt{B}}\right)\right|^{2} \tag{36}$$

which seems a new result (not reported in the literature before).

Especially, when $m=0$, (36) reduce to the HF with time evolution of squeezed vacuum state

$$\mathcal{H}_{m=0}(p,q;t)=Ce^{D}\operatorname{sech}r\left|\sum_{l=0}^{\infty}\frac{(-Be^{-2\kappa t}\tanh r)^{l}}{l!2^{l}}H_{2l}\left(\frac{A}{2i\sqrt{B}}\right)\right|^{2} \tag{37}$$

when $\kappa t=0$, $T=0$, (36) becomes the HF of PASVS

$$\mathcal{H}(p,q;0)=N_{r,m}^{2}Ce^{D}\operatorname{sech}r\,B^{m}\left|\sum_{l=0}^{\infty}\frac{(-B\frac{\tanh r}{2})^{l}}{l!}H_{2l+m}\left(\frac{A}{2i\sqrt{B}}\right)\right|^{2} \tag{38}$$

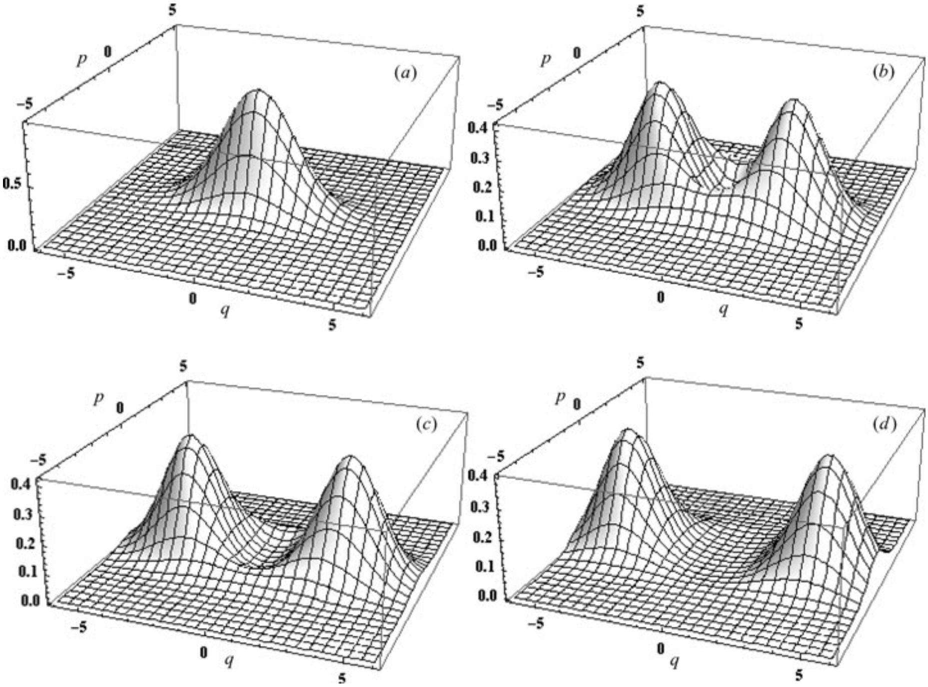


**Fig. 2** HF of PASVS with $r = 0.5$ and $g = 0.5$ fixed at $\kappa t = 0.05$ for several different added-photon number: **(a)** $m = 0$, **(b)** $m = 2$, **(c)** $m = 3$, and **(d)** $m = 5$, respectively

while $\kappa t \to \infty$, $T \to 1$, then

$$\mathcal{H}(p, q; \infty) = Ce^D = \frac{2\sqrt{g}}{1+g} \exp\left(-\frac{gq^2 + p^2}{1+g}\right) \tag{39}$$

which is also a Gaussian form.

In order to see the behavior of HF evolution of PASVS, we draw the three-dimensional graphics of $\mathcal{H}(p, q; t)$ in Figs. 1–3 as well. It is clear seen from Fig. 1 that HF distribution becomes from non-Gaussian to Gaussian after coupling to the environment, which is similar to WF case, but here non-Gaussian distribution is fully positive. In addition, Figs. 2 and 3 present how the variation of added-photon number $m$ and Gaussian spatial width parameter $g$ affects the PASVS's HF for a given evolution time $\kappa t$.

## 5 Conclusions

In summary, we have introduced the effect of decoherence on the PASVS in dissipative channel by virtue of describing the time evolution of its HF. Firstly, we derive the normalization factor of PASVS in the form of Legendre polynomial of the squeezing parameter. After obtaining the normal ordered form of PASVS's density operator in dissipative channel, we give the explicit analytical expression of HF and discuss its time evolution in dissipative channel. The merit of doing so lies in the fact that HF can avoid the appearance of the negativity of WF. Finally, it is worth mentioning that when an excited atom passes through a cavity field

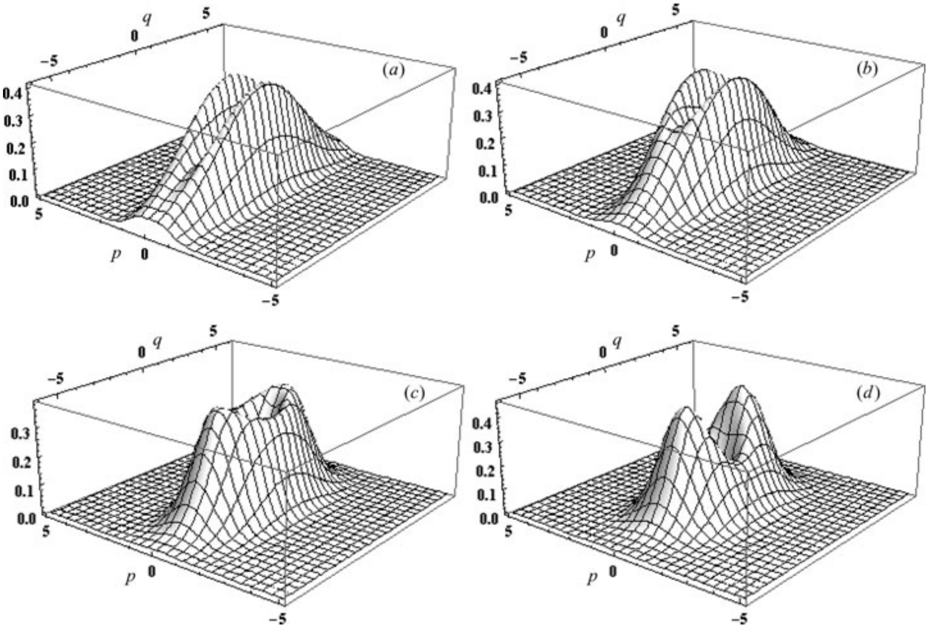


**Fig. 3** HF of PASVS with $m = 1$ and $r = 0.5$ fixed at $\kappa t = 0.05$ for several different Gaussian spatial width parameter: **(a)** $g = 0.1$, **(b)** $g = 0.2$, **(c)** $g = 0.4$, and (d) $g = 0.8$, respectively

which is in a squeezed vacuum state, then their interaction may produce photon excitation on the squeezed vacuum state, i.e. PASVS.

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